

## EXPRESSIONS AS STATEMENTS

A **numerical expression** uses numerals and operation signs to represent a value. An **algebraic expression** uses numerals, operation signs, and variables to represent a value.

Word Expression	Numerical Expression	Word Expression	Algebraic Expression
Eight more than seventeen	$17 + 8$	Marie's age four years ago	$m - 4$ ( $m$ is her age today)
Four times the difference between six and ten	$4 \times (10 - 6)$	Each share of the cost of dinner split evenly among three friends	$d \div 3$ ( $d$ is the total cost of the dinner)
Twice the sum of eight and five	$2 \times (8 + 5)$	Twenty more than the product of 6 and a number	$(6 \times n) + 20$ ( $n$ is the number)

Write a numerical expression to represent each word expression.

1. four less than the product of 7 and 9 _____	2. the quotient of 100 and 5, increased by 3 _____	3. six times the sum of 19 and 2 _____
4. 9 squared divided by 3 _____	5. 8 times the product of 3 and 6 _____	6. half of 42, increased by 5 _____

Write an algebraic expression to represent each word expression. Let  $n$  represent the number referred to each time.

7. 10 less than the sum of a number and 25 _____	8. the difference between 19 and 6, increased by a number _____	9. twice a number, decreased by 4 _____
10. 7 more than the square root of a number _____	11. a number divided by 9 _____	12. one-third of a number, increased by 8 _____

## USES OF VARIABLES

A **variable** is a symbol that represents a number. Sometimes variables represent a specific number. Other times variables represent a range of values.

### Variable Represents a Specific Value

$$m + 30 = 105$$

$$m + 30 - 30 = 105 - 30$$

$$m = 75$$

There is only one value for  $m$ .

### Variable Represents a Range of Values

$$b + 8 > 17$$

$$b + 8 - 8 > 17 - 8$$

$$b > 9$$

The value of  $b$  is any number greater than 9.

Find the value of the variable.

1.  $c + 15 = 29$

\_\_\_\_\_

2.  $k - 7 = 18$

\_\_\_\_\_

3.  $f - 16 = 16$

\_\_\_\_\_

4.  $8y = 120$

\_\_\_\_\_

5.  $1.5x = 7.5$

\_\_\_\_\_

6.  $w \div 9 = 75$

\_\_\_\_\_

Find the range of values that makes each inequality true.

7.  $a + 4 < 10$

\_\_\_\_\_

8.  $c - 15 > 3$

\_\_\_\_\_

9.  $p + 7 > 31$

\_\_\_\_\_

10.  $d - 54 < 19$

\_\_\_\_\_

11.  $s + 22 > 55$

\_\_\_\_\_

12.  $g + 29 > 101$

\_\_\_\_\_

13.  $36 + a < 47$

\_\_\_\_\_

14.  $s - 16 > 34$

\_\_\_\_\_

15.  $y + 92 < 157$

\_\_\_\_\_

16.  $25 + f > 91$

\_\_\_\_\_

17.  $r + 123 > 205$

\_\_\_\_\_

18.  $z - 79 < 103$

\_\_\_\_\_

19.  $3s < 27$

\_\_\_\_\_

20.  $1.2y > 8.4$

\_\_\_\_\_

21.  $t \div 8 < 19$

\_\_\_\_\_

### Challenge

Write an inequality for which the answer is  $k < 37$ .

\_\_\_\_\_

## REPRESENTATION OF PATTERNS AND FUNCTIONS

A **function** is a relationship between or among numbers. In one type of function, the value of one number, or the **independent variable**, determines the value of another number, or the **dependent variable**.

Members of a garden club are selling plants to raise money for a town park. The club receives \$35 for every batch of 20 plants its members sell. Complete the table below to determine how much the club raises for given numbers of plants it will sell.

Number of Plants Sold	Amount Raised
20	\$35.00
40	\$70.00
60	1. _____
80	2. _____
100	3. _____

Number of Plants Sold	Amount Raised
120	4. _____
140	5. _____
160	6. _____
180	7. _____
200	8. _____

9. What is the independent variable? \_\_\_\_\_
10. What is the dependent variable? \_\_\_\_\_
11. How many plants must club members sell to raise at least \$250.00, their goal?

\_\_\_\_\_

\_\_\_\_\_

Don earns \$12.75 for each lawn he mows. Complete the table to determine how much he earns for mowing any given number of lawns.

Number of Lawns Mowed	Amount Earned
1	\$12.75
2	\$25.50
3	12. _____
4	13. _____
5	14. _____

Number of Lawns Mowed	Amount Earned
6	15. _____
7	16. _____
8	17. _____
9	18. _____
10	19. _____

20. How many lawns must Don mow to earn at least \$100? \_\_\_\_\_

## DEFINING FUNCTIONS

A **function** can be represented by an equation that describes how a change in one or more variables results in a change in another variable.

$y = 4x - 2$  ← Function expressed as an equation

- If  $x$  is 1, then  $y$  is  $4(1) - 2$  or 2
- If  $x$  is 0, then  $y$  is  $4(0) - 2$  or  $-2$
- If  $x$  is  $-1$ , then  $y$  is  $4(-1) - 2$  or  $-6$

Complete the tables to show values of  $x$  and  $y$  in each function.

1.  $y = 2x + 3$

x	y
1	_____
0	_____
-1	_____

2.  $y = -4 + x$

x	y
3	_____
2	_____
0	_____

3.  $y = 3x - 1$

x	y
2	_____
0	_____
-2	_____

4.  $x = 5y - 4$

x	y
_____	2
_____	1
_____	-1

5.  $x = 10 - 2y$

x	y
_____	2
_____	-1
_____	-5

6.  $x = y - 1$

x	y
_____	4
_____	0
_____	-4

7.  $y = 2x - 5$

x	y
4	_____
0	_____
-4	_____

8.  $y = -x + 3$

x	y
2	_____
0	_____
-1	_____

9.  $y = 6 + x$

x	y
-1	_____
0	_____
1	_____

10.  $x = 4y + 2$

x	y
_____	-4
_____	0
_____	4

11.  $x = y - \frac{3}{4}$

x	y
_____	$\frac{1}{4}$
_____	$\frac{3}{4}$
_____	$-1\frac{1}{2}$

12.  $x = 1.5y - 8$

x	y
_____	0
_____	8
_____	-6

## SOLVING LINEAR EQUATIONS

Find the value of  $y$  in the equation  $2y + 6 = 14$ . To solve linear equations, use inverse operations to isolate the variable on one side of the equation—use subtraction and addition with each other, and use multiplication and division with each other.

$$2y + 6 = 14$$

$$2y + 6 - 6 = 14 - 6 \leftarrow \text{Add } -6 \text{ to each side of the equation.}$$

$$2y = 8$$

$$\frac{2y}{2} = \frac{8}{2} \leftarrow \text{Divide each side of the equation by 2.}$$

$$y = 4$$

Solve.

1.  $3a - 9 = 27$

\_\_\_\_\_

2.  $6n + 10 = -44$

\_\_\_\_\_

3.  $12 + 7c = 103$

\_\_\_\_\_

4.  $\frac{h}{5} + 3 = -6$

\_\_\_\_\_

5.  $\frac{x}{8} + 2 = 14$

\_\_\_\_\_

6.  $9d - 7 = 74$

\_\_\_\_\_

7.  $\frac{w}{-11} + 8 = 5$

\_\_\_\_\_

8.  $\frac{i}{9} - 4 = -23$

\_\_\_\_\_

9.  $2y - 6 = -34$

\_\_\_\_\_

10.  $-18 + 6b = 72$

\_\_\_\_\_

11.  $-10m + 9 = 59$

\_\_\_\_\_

12.  $\frac{t}{-6} + 15 = -45$

\_\_\_\_\_

13.  $2b - 18 = -6$

\_\_\_\_\_

14.  $\frac{t}{3} + 4 = -5$

\_\_\_\_\_

15.  $22 - 5c = -8$

\_\_\_\_\_

16.  $\frac{d}{-5} - 6 = 20$

\_\_\_\_\_

17.  $-8p + 9 = 25$

\_\_\_\_\_

18.  $6x - 19 = 89$

\_\_\_\_\_

19.  $\frac{n}{4} - 12 = -8$

\_\_\_\_\_

20.  $-15 - 3f = -51$

\_\_\_\_\_

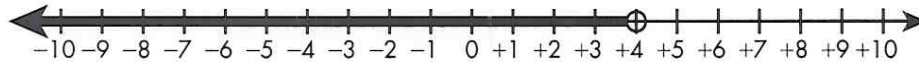
21.  $\frac{m}{-3} + 24 = -6$

\_\_\_\_\_

# SOLVING INEQUALITIES AND NON-LINEAR EQUATIONS

You can graph the solution set of an inequality on a number line.

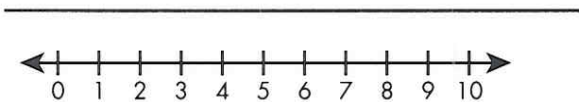
$$\begin{aligned}
 3x + 9 &< 21 \\
 3x + 9 - 9 &< 21 - 9 \\
 \frac{3x}{3} &< \frac{12}{3} \\
 x &< 4
 \end{aligned}$$



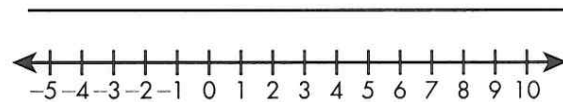
The open circle shows that 4 is not part of the solution set since  $x$  is less than 4. If 4 were part of the solution set, then the graph would have a closed (filled) circle at 4.

**Solve each inequality. Graph the solution set.**

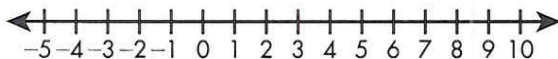
1.  $2c + 5 \geq 17$



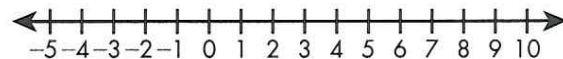
2.  $12 + 7a \leq 26$



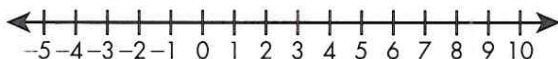
3.  $13 < 4k - 3$



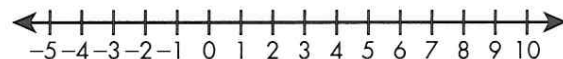
4.  $9s + 8 > 35$



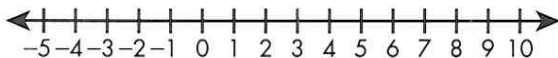
5.  $6w + 8 \leq 56$



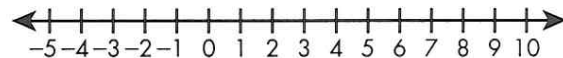
6.  $11k + 12 > 67$



7.  $17b - 14 \geq 37$



8.  $14j + 3 \leq 59$



## SPECIAL VALUES OF PATTERNS, RELATIONSHIPS, AND FUNCTIONS

The point where a line crosses the  $x$ -axis of a graph is called the  **$x$ -intercept**. The point where a line crosses the  $y$ -axis of a graph is called the  **$y$ -intercept**. You can use the intercepts of a line to graph the equation.

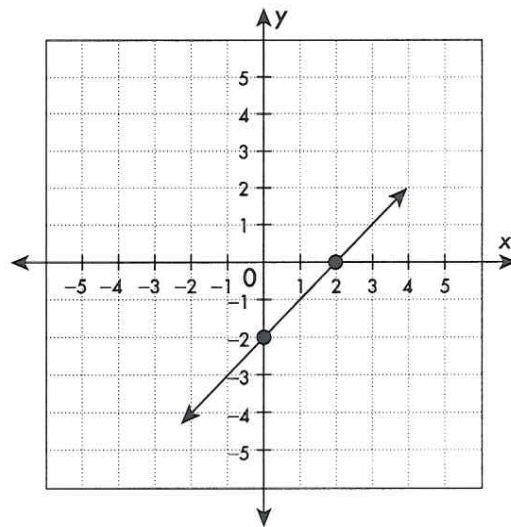
**Graph the equation:  $2x - y = 4$ .**

Find the  $x$ -intercept. Substitute 0 for  $y$ .  
Then solve for  $x$ .

$$\begin{aligned} 2x - 0 &= 4 \\ \frac{2x}{2} &= \frac{4}{2} \\ x &= 2 \quad \leftarrow \text{The } x\text{-intercept is } 2. \end{aligned}$$

Find the  $y$ -intercept. Substitute 0 for  $x$ .  
Then solve for  $y$ .

$$\begin{aligned} 2(0) - y &= 4 \\ 0 - y &= 4 \\ \frac{-y}{-1} &= \frac{4}{-1} \\ y &= -4 \quad \leftarrow \text{The } y\text{-intercept is } -4. \end{aligned}$$



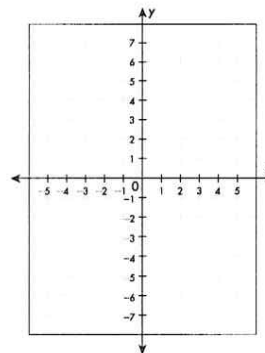
Plot the points  $(2, 0)$  and  $(0, -4)$  on the graph. Then draw a line connecting these points.

**Find the  $x$ - and  $y$ -intercepts. Then graph the equation.**

**1.**  $3x + y = 6$

$x$ -intercept \_\_\_\_\_

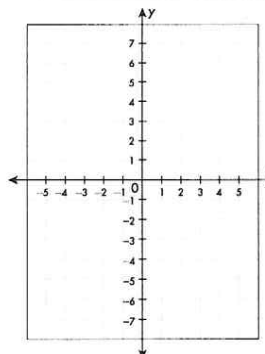
$y$ -intercept \_\_\_\_\_



**2.**  $2x - y = 1$

$x$ -intercept \_\_\_\_\_

$y$ -intercept \_\_\_\_\_



## BASIC OPERATIONS ON ALGEBRAIC EXPRESSIONS

An algebraic term contains a number and a variable. The number is called the **coefficient** of the variable. You can combine like terms or terms that have the same variable.

**Simplify.**  $8a + 6b - 3a + 5b$

- Combine the terms containing the variable  $a$ .  $8a - 3a = (8 - 3)a = 5a$
- Combine the terms containing the variable  $b$ .  $6b + 5b = (6 + 5)b = 11b$
- Rewrite the expression.  $5a + 11b$

**Simplify.**

1. $11c + 5d - 6c + 2c$ _____	2. $9a - 11b - 4b + 7a$ _____	3. $7(2n - 3r) - 8r$ _____
4. $-5(m + 2s) + 9m$ _____	5. $8(-4b + 2d) - 12d + 2b$ _____	6. $4(j + k) + -5(j - k)$ _____
7. $12f - 6g - 12g - 7f$ _____	8. $-3(11t - 5f) + 4t$ _____	9. $2m - 9p + 8p - 3m$ _____
10. $3(9c - 2a) - 4(2c + 5a)$ _____	11. $7n - 5(n - s) + 4s$ _____	12. $9k - 6j - 9(2k + j)$ _____
13. $7(r + 2s) - 15r + 5s$ _____	14. $-3(p - 4q) + 4(-2q)$ _____	15. $\frac{3}{4}(a - 8b) - \frac{1}{2}a + 6b$ _____
16. $8m - 6(3m - 4n)$ _____	17. $12f + \frac{5}{8}(8g) - 10f$ _____	18. $-3(x + \frac{5}{12}y) - 7x + 4y$ _____



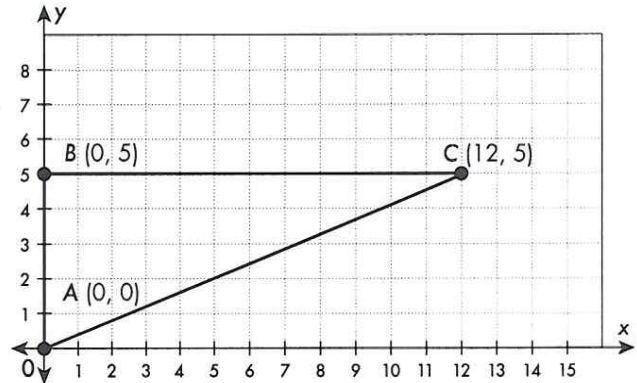
## RECTANGULAR COORDINATE SYSTEM FOR PROBLEMS

An airline navigator has plotted the location of 3 airports on a coordinate grid. The airports are labeled A, B, and C. The distance between adjacent grid lines is 1 mile. What is the distance between airports A and C?

You can use the Pythagorean Theorem to find distances on a coordinate grid.

- Points A and B lie on a single vertical line.  
Count the units to find the length of  $\overline{AB}$ . \_\_\_\_\_
- Points B and C lie on a single horizontal line.  
Count the units to find the length of  $\overline{BC}$ . \_\_\_\_\_
- Use the Pythagorean Theorem to find the length of  $\overline{AC}$ .

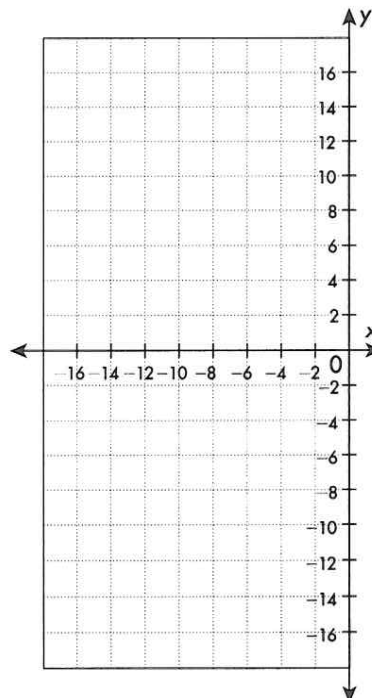
$$\begin{aligned} AB^2 + BC^2 &= AC^2 \\ 5^2 + 12^2 &= AC^2 \\ 25 + 144 &= AC^2 \\ 169 &= AC^2 \\ 13 &= AC \end{aligned}$$



- The distance between airports A and C is \_\_\_\_\_

On a navigator's grid, the distance between adjacent grid lines is 1 mile. A plane departed from an airport located at  $(-11, -12)$  and flew to  $(-11, 12)$  and then made a right turn and landed at  $(-4, 12)$ . How far was the plane from its starting point?

- Plot and label the points on the coordinate grid.
- What is the distance from  $(-11, -12)$  to  $(-11, 12)$ ?  
\_\_\_\_\_
- What is the distance from  $(-11, 12)$  to  $(-4, 12)$ ?  
\_\_\_\_\_
- How far was the plane from its starting point? \_\_\_\_\_



## GRAPHIC SOLUTIONS OF SIMPLE SYSTEMS OF EQUATIONS

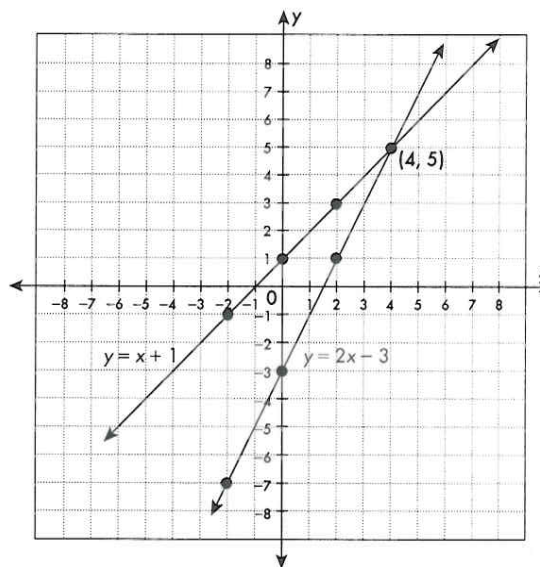
A **system of equations** is two or more equations to be solved for values of the variables satisfying both or all of them. The equations can be graphed on the same coordinate plane. The point where the graphs of the equations intersect represents a **solution**.

**What values of  $x$  and  $y$  satisfy both of these equations?  $y = 2x - 3$ ;  $y = x + 1$**

- Make a table of ordered pairs for each equation.

$y = 2x - 3$		$y = x + 1$	
$x$	$y$	$x$	$y$
-2	-7	-2	-1
0	-3	0	1
2	1	2	3

- Write the ordered pairs for each equation.  
 $y = 2x - 3$ :  $(-2, -7)$ ,  $(0, -3)$ ,  $(2, 1)$   
 $y = x + 1$ :  $(-2, -1)$ ,  $(0, 1)$ ,  $(2, 3)$
- Graph the ordered pairs on the same coordinate plane. Find the point where the lines intersect. This is the solution of the system.

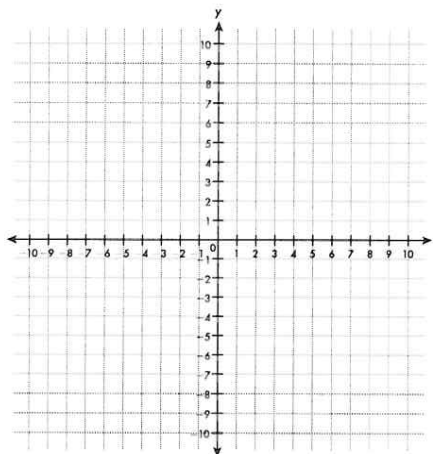
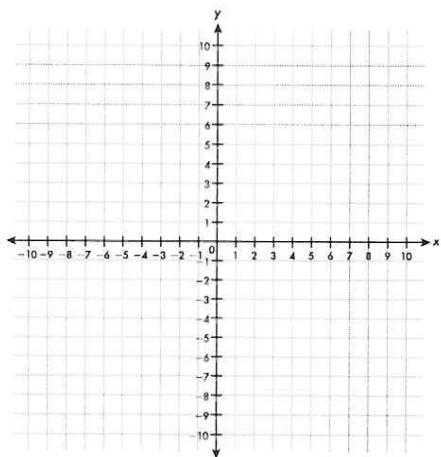


The lines intersect at  $(4, 5)$ . So,  $(4, 5)$  is the solution of the system.

**Solve the system of equations by graphing.**

1.  $y = x + 2$  and  $y = 6 - x$

2.  $y = 2x + 1$  and  $y = 4 - x$



## PROPERTIES OF ARITHMETIC AND GEOMETRIC SEQUENCES

An **arithmetic sequence** is made up of terms that change by the same amount. A **geometric sequence** is made up of terms that change by the same ratio.

9, 4, -1, -6, -11, ... ← Arithmetic Sequence → Add -5 to a term to get to the next term.

500, 250, 125, 62.5, ... ← Geometric Sequence → Each term is half of the previous term.

**Find the next two terms in each arithmetic sequence. State the rule used to form the sequence.**

1. -37, -29, -21, -13, \_\_\_\_\_, \_\_\_\_\_, ... \_\_\_\_\_
2. 28, 15, 2, \_\_\_\_\_, \_\_\_\_\_, ... \_\_\_\_\_
3. 157, 128, 99, 70, \_\_\_\_\_, \_\_\_\_\_, ... \_\_\_\_\_
4. -14, -31, -48, -65, \_\_\_\_\_, \_\_\_\_\_, ... \_\_\_\_\_
5. -25, -16, -7, 2, \_\_\_\_\_, \_\_\_\_\_, ... \_\_\_\_\_
6. -12.5, -12.0, -11.5, -11, \_\_\_\_\_, \_\_\_\_\_, ... \_\_\_\_\_
7. -18, -8, 2, 12, \_\_\_\_\_, \_\_\_\_\_, ... \_\_\_\_\_

**Find the next two terms in each geometric sequence. State the rule used to form the sequence.**

8. 400, 200, 100, 50, \_\_\_\_\_, \_\_\_\_\_, ...  
\_\_\_\_\_
9. -9, 27, -81, 243, \_\_\_\_\_, \_\_\_\_\_, ... \_\_\_\_\_
10. 18, 45, 112.5, 281.25, \_\_\_\_\_, \_\_\_\_\_, ... \_\_\_\_\_
11. 80,000; 20,000, 5,000, 1,250, \_\_\_\_\_, \_\_\_\_\_, ...  
\_\_\_\_\_
12. 17, -187, 2,057, -22,627, \_\_\_\_\_, \_\_\_\_\_, ... \_\_\_\_\_
13. 15, -7.5, 3.75, -1.875, \_\_\_\_\_, \_\_\_\_\_, ...  
\_\_\_\_\_
14. 25, 45, 81, 145.8, 262.44, \_\_\_\_\_, \_\_\_\_\_, ... \_\_\_\_\_